Irrational Numbers Exam

A. Simplify the following:

1. $\sqrt{64x^2} = \sqrt{2^6 x^2} = 2^3 x$

2. $\sqrt{27x^3} = \sqrt{3^3 x^3} = 3x\sqrt{3x}$

3. $\sqrt{2016} = \sqrt{2^5 \cdot 3^3 \cdot 7} = 2^2 \cdot 3\sqrt{2 \cdot 7}$

4. $\sqrt{5x^7 y^{12}} = x^3 y^6 \sqrt{5x}$

5. $3\sqrt{7} \bullet 5\sqrt{11} = 3 \cdot 5 \cdot \sqrt{7 \cdot 11} = 15\sqrt{77}$

6. $2\sqrt{15} \bullet 3\sqrt{45} = 2\sqrt{3 \cdot 5} \cdot 3\sqrt{3^2 \cdot 5} = 2 \cdot 3\sqrt{3^3 \cdot 5^2} = 2 \cdot 3 \cdot 3 \cdot 5\sqrt{3} = 90\sqrt{3}$

7. $\sqrt{xy^3} \bullet \sqrt{x^5 y^4} = \sqrt{x^6 y^7} = x^3 y^3 \sqrt{y}$

8. $3x\sqrt{2x^3 y^5} \bullet 2y\sqrt{8x^3 y^4} = 3x\sqrt{2x^3 y^5} \bullet 2y\sqrt{2^3 x^3 y^4} = 3x \cdot 2y\sqrt{2^4 x^6 y^9} =$
$3x \cdot 2y \cdot 2^2 x^3 y^4 \sqrt{y} = 3 \cdot 2^3 x^4 y^5 \sqrt{y} = 24x^4 y^5 \sqrt{y}$

9. $4xz\sqrt{x^4 y^3} \bullet 5y^2\sqrt{18x^3 y^5 z^6} = 4xz\sqrt{x^4 y^3} \bullet 5y^2\sqrt{2 \cdot 3^2 x^3 y^5 z^6} = 4xz \cdot 5y^2\sqrt{2 \cdot 3^2 x^7 y^8 z^6} =$
$4xz \cdot 5y^2 \cdot 3x^3 y^4 z^3 \sqrt{2x} = 60x^4 y^6 z^4 \sqrt{2x}$

10. $2\sqrt{3}\left(4 + \sqrt{2}\right) = 2\sqrt{3} \cdot 4 + 2\sqrt{3} \cdot \sqrt{2} = 8\sqrt{3} + 2\sqrt{6}$

11. $\left(3 - 5\sqrt{3}\right)^2 = \left(3 - 5\sqrt{3}\right)\left(3 - 5\sqrt{3}\right) = 3\left(3 - 5\sqrt{3}\right) - 5\sqrt{3}\left(3 - 5\sqrt{3}\right) =$
$9 - 15\sqrt{3} - 15\sqrt{3} + 25\sqrt{3^2} = 9 - 30\sqrt{3} + 25 \cdot 3 = 9 - 30\sqrt{3} + 75 = 84 - 30\sqrt{3}$

12. $\left(2 + 3\sqrt{7}\right)\left(5 - \sqrt{3}\right) = 2\left(5 - \sqrt{3}\right) + 3\sqrt{7}\left(5 - \sqrt{3}\right) = 10 - 2\sqrt{3} + 15\sqrt{7} - 3\sqrt{21}$

13. $\dfrac{5}{\sqrt{3}} = \dfrac{5}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{5\sqrt{3}}{3}$

14. $\dfrac{3\sqrt{2}}{5\sqrt{5}} = \dfrac{3\sqrt{2}}{5\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{3\sqrt{6}}{5 \cdot 5} = \dfrac{3\sqrt{6}}{25}$

15. $\dfrac{3 + 2\sqrt{7}}{4\sqrt{5}} = \dfrac{\left(3 + 2\sqrt{7}\right)}{4\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{5} \cdot 3 + \sqrt{5} \cdot 2\sqrt{7}}{4 \cdot 5} = \dfrac{3\sqrt{5} + 2\sqrt{35}}{20}$

16. $2\sqrt{x} + 4\sqrt{x} - 9\sqrt{x} = -3\sqrt{x}$

17. $3\sqrt{5} - 2\sqrt{7} + 6\sqrt{5} + 11\sqrt{7} = 9\sqrt{5} + 9\sqrt{7}$

18. $6\sqrt{27}-8\sqrt{75}=6\sqrt{3^3}-8\sqrt{3\cdot 5^2}=6\cdot 3\sqrt{3}-8\cdot 5\sqrt{3}=18\sqrt{3}-40\sqrt{3}=-22\sqrt{3}$

19. $3x\sqrt{5y^3}-2y\sqrt{125x^2y}=3x\sqrt{5y^3}-2y\sqrt{5^3x^2y}=3x\cdot y\sqrt{5y}-2y\cdot 5x\sqrt{5y}=$
$3xy\sqrt{5y}-10xy\sqrt{5y}=-7xy\sqrt{5y}$

20. $\dfrac{5}{2+\sqrt{6}}=\dfrac{5}{\left(2+\sqrt{6}\right)}\cdot\dfrac{\left(2-\sqrt{6}\right)}{\left(2-\sqrt{6}\right)}=\dfrac{10-5\sqrt{6}}{4-6}=\dfrac{10-5\sqrt{6}}{-2}$

21. $\dfrac{4\sqrt{6}}{2-3\sqrt{3}}=\dfrac{4\sqrt{6}}{\left(2-3\sqrt{3}\right)}\cdot\dfrac{\left(2+3\sqrt{3}\right)}{\left(2+3\sqrt{3}\right)}=\dfrac{8\sqrt{6}+12\sqrt{18}}{4-9\cdot 3}=\dfrac{8\sqrt{6}+12\sqrt{2\cdot 3^2}}{4-27}=$
$\dfrac{8\sqrt{6}+12\cdot 3\sqrt{2}}{-23}=\dfrac{8\sqrt{6}+36\sqrt{2}}{-23}$